FLOW OF GAS IN THE CHANNELS OF A CYLINDRICAL SOLID, FREELY MOVING BETWEEN END CHAMBERS

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A numerical solution is derived for the heat transfer and hydraulic resistance of transient gas flows in the channels of a cylindrical solid (such as a piston or ejector) moving freely between end chambers.

Devices incorporating elements moving freely through channels are widely employed in industrial processes; they include pneumatic regulating systems, pneumatic conveyers, and so forth. Methods of calculating such systems based on a simultaneous consideration of their thermal gasdynamic and kinematic characteristics have been developed by a number of authors [1-3], but in the cases so far the gas flow in the channels of the freely-moving solids did not have any major influence on the working process and was accordingly not taken into account. At the same time there are certain technical devices (Fig. 1, [4]) containing a "floating" piston (or ejector), incorporating channels with a regenerative packing of considerable heat capacity. In this case the heat transfer and resistance of the transient gas flows relative to the packing material in the moving solid play a substantial role. A method of calculating the thermal gasdynamics in chambers communicating via a complex pipeline was developed in [5], but the law governing the motion of the piston (ejector) was there determined by the kinematic arrangement of the system under consideration.

In this paper we shall propose a method of calculating the thermal gasdynamic and kinematic characteristics of systems including elements moving freely between end chambers. For the mathematical description of the problem we made the following assumptions: 1) The working gas satisfies the equation of state of an ideal gas, the specific heat of the gas being constant; 2) conductive heat transfer in the gas flow is negligibly slight by comparison with convective heat transfer; 3) the thermal conductivity of the packing is equal to zero along the gas flow and is infinitely great at right angles to the latter; 4) the temperature drop across the thickness of the chamber and channel walls may be neglected, since the relationship $Bi = \alpha\delta/\lambda \ll 1$ is satisfied.

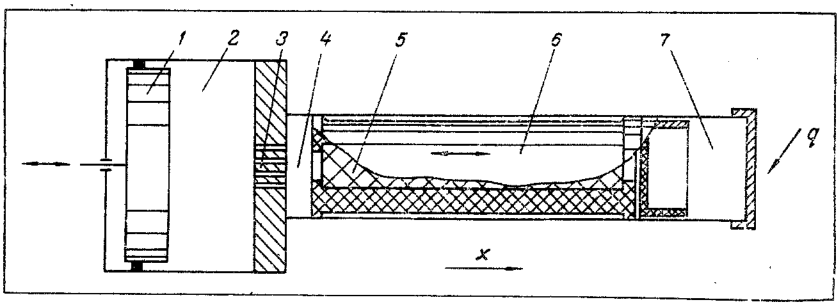

Fig. 1. Arrangement of the cryogenic machine: 1) Piston; 2) compressor space; 3) water cooler; 4) hot space; 5) regenerator; 6) freely-moving ejector; 7) cold space.

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Thermal gasdynamic processes in channels are described by a system of one-dimensional differential equations in partial derivatives for parameters averaged over the cross section of the flow [6], which in the inertial coordinate system takes the form:

$$\frac{\partial}{\partial t} F\rho U + \frac{\partial}{\partial x_c} \beta_1 F\rho U^2 = -F \frac{\partial P}{\partial x_c} - \frac{\zeta}{2d_h} F\rho U_g^2; \tag{1}$$

$$\frac{\partial}{\partial t} F\rho + \frac{\partial}{\partial x_c} (F\rho U) = 0; \tag{2}$$

$$\frac{\partial}{\partial t}\left[F\rho\left(i + \beta_1 \frac{U^2}{2} - \frac{P}{\rho}\right)\right] + \frac{\partial}{\partial x_c}\left[F\rho U\left(i + \alpha_1 \frac{U^2}{2}\right)\right] = -\sum_{\varepsilon = p, w} \frac{4\alpha_\varepsilon}{d_h} F_\varepsilon \left(T_g - T_\varepsilon + \frac{U_g^2}{2c_r}\right), \tag{3}$$

where $U = U_g + U_e$ is the absolute velocity of the gas, U_g is the velocity of the gas relative to the ejector, and U_e is the ejector velocity.

$$\rho_w\left(\frac{\partial e_w}{\partial t} + U_e \frac{\partial e_w}{\partial x_w}\right) = \frac{\alpha}{\delta_w}\left(T_g - T_w + \frac{U_g^2}{2c_r}\right) + \frac{\alpha_a}{\delta_w}(T_a - T_w); \tag{4}$$

$$\rho_p\left(\frac{\partial e_p}{\partial t} + U_e \frac{\partial e_p}{\partial x_c}\right) = \frac{4\alpha}{d_{hp}} \cdot \frac{p}{1-p}\left(T_g - T_p + \frac{U_g^2}{2c_r}\right); \tag{5}$$

$$P = \rho R T_g. \tag{6}$$

The coefficients α_1, β_1 will be equated to unity in subsequent analysis.

The processes taking place in the chambers are described by a system of ordinary differential equations for concentrated parameters (the gas flow is regarded as positive when the chamber is being filled):

$$\frac{d}{dt}(m_n e_n) = -P_n \frac{dV_n}{dt} + \sum_{i = r, T} G_{ni} J_{ni} - F_{cn}\alpha_n (T_{gn} - T_{wn}) + q_n; \tag{7}$$

$$\rho_{wn}\delta_{wn} \frac{de_{wn}}{dt} = \alpha_n (T_{gn} - T_{wn}) + \alpha_{an}(T_{an} - T_{wn}) + q_{wn}; \tag{8}$$

$$\frac{dm_n}{dt} = \sum_{i = r, T} G_{ni}; \tag{9}$$

$$P_n V_n = m_n R T_{gn}; \tag{10}$$

$$\frac{dV_n}{dt} = \omega F_e \frac{dS}{dt}; \quad \omega = 1 \text{ for } n = 1; \ \omega = -1 \text{ for } n = 2; \tag{11}$$

$$\frac{dS}{dt} = U_e; \quad m_e \frac{dU_e}{dt} = F_e (P_1 - P_2) - \Sigma P, \tag{12}$$

where ΣP is the set of deleterious forces of resistance, the weight of the forward-moving elements, and so on,

$$f_n = f_n(t). \tag{13}$$

Gas exchange with the receiver (if the system contains one) is determined from the condition of quasi-steady-state gas flow between the receiver and the chambers [7].

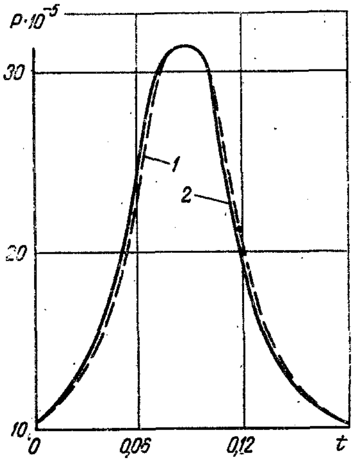

Fig. 2. Change in pressure during the working cycle: 1) Cold chamber; 2) hot chamber. P in N/m^2, t in sec.

The conditions for the conjugation of the chambers with the channels [5, 6] are:

at the entrance into the channel (x = 0)

$$P_n = P_0 + \zeta_0\rho_0 U_0^2; \quad \frac{2k}{k-1} RT_{gn} = \frac{2k}{k-1} RT_{g0} + U_0^2; \tag{14}$$

at the exit from the channel (x = 1)

$$P_n = P_l + \frac{F_l}{F_n}\zeta_l\rho_l U_l^2. \tag{15}$$

As initial conditions we take an arbitrary distribution of the parameters in the system under consideration. The difference approximation to the system of equations (1)-(13) and the conjugation conditions for a unidirectional flow of gas in the channels (implicit scheme) takes the following form, after making a few transformations in the original equations. Channels (after transforming to a coordinate system attached to the ejector):

$$U_{g,t} + U_{e,t} + U_g U_{g,x} + R\frac{T_g}{P} P_{,x} = -\frac{\zeta}{2d_h}\hat{U}_g U_g; \tag{16}$$

$$P_{,t} + U_g P_{,x} - \frac{P}{T_g}(T_{g,t} + U_g T_{g,x}) + PU_{g,x} = 0; \tag{17}$$

$$T_{g,t} + U_g T_{g,x} - \frac{k-1}{k}\cdot\frac{T_g}{P}(P_{,t} + U_g P_{,x}) =$$

$$= -\sum_{\varepsilon=p,w}\frac{4\alpha_\varepsilon}{d_{h\varepsilon}}\cdot\frac{k-1}{k}\,\frac{T_g}{P}\left(\hat{T}_g - T_\varepsilon + \frac{U_g^2}{2c_r}\right) + \frac{\zeta}{2d_h} : \frac{U_g^2 U}{c_r}; \tag{18}$$

$$T_{w,t} = \frac{1}{\rho_w c_w}\left[\frac{\alpha}{\delta_w}\left(\hat{T}_g - \hat{T}_w + \frac{U_g^2}{2c_r}\right) + \frac{\alpha_a}{\delta_w}(T_a - \hat{T}_w)\right]; \tag{19}$$

$$T_{p,t} = \frac{1}{\rho_p c_p}\cdot\frac{4\alpha_p}{d_{hp}}\cdot\frac{p}{1-p}\left(\hat{T}_g - \hat{T}_p + \frac{U_g^2}{2c_r}\right). \tag{20}$$

Chambers (index n provisionally omitted):

$$T_{g,t} = -(k-1)\hat{T}_g\frac{1}{V}V_{,t} + \frac{1}{m}\sum_i \hat{G}_i(kT_{gi}^* - T_g) -$$

$$-\frac{A}{m}\hat{T}_g + \frac{A}{m}T_w + \frac{q}{c_v m}; \tag{21}$$

$$P_{,t} = -k\hat{P}\frac{1}{V}V_{,t} + \frac{R}{V}\sum_i \hat{G}_i kT_{gi}^* -$$

$$-\frac{A}{m}\hat{P} + \frac{AR}{V}T_w + \frac{qR}{c_v V}; \tag{22}$$

$$m_{,t} = \sum_i \hat{G}_i\beta, \tag{23}$$

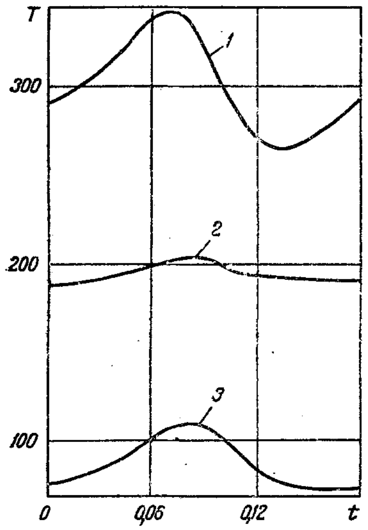

Fig. 3

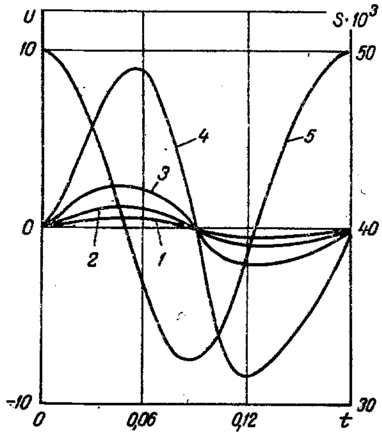

Fig. 4

Fig. 3. Temperature diagram: 1) Compressor chamber; 2) median section of the regenerator; 3) cold chamber. T in °K, t in sec.

Fig. 4. Cyclic change of parameters in the machine: 1) Velocity of the ejector; 2), 3) gas velocity at the cold and hot ends of the regenerator respectively; 4) average gas velocity in the cooling system (taken with respect to length); 5) motion of the ejector in the cold chamber. U in m/sec, S in m, t in sec.

where

$$A = \frac{4\alpha}{dc_v}\left(\frac{\pi}{8}d^3 + V\right); \quad \hat{G}_i = \begin{cases} \hat{G}_r \text{ — gas exchange with the receiver} \\ \hat{U}_{g\tau}\rho_\tau F_\tau \text{ — gas exchange with the channel (tube)} \end{cases}$$

when the chamber is being filled

$$\beta = 1; \quad T^*_{gi} = \begin{cases} T_{gr} \text{ — gas exchange with the receiver} \\ T_{g\tau} + \hat{U}^2_{g\tau}/2c_r \text{ — gas exchange with the channel (tube)} \end{cases}$$

when gas is flowing out from the chamber

$$T^*_{gi} = T_g; \quad \beta = \begin{cases} \hat{m}/m \text{ — outflow into the receiver} \\ 1 \text{ — outflow into the channel} \end{cases}$$

The gas flow $\hat{G}_r$ is determined from the values of the parameters in the preceding time layer. The mesh functions $f_n = \Phi(i\tau)$ are usually known.

$$T_{w,t} = \frac{1}{\delta_w \rho_w c_w}\left[\alpha(\hat{T}_g - \hat{T}_w) + \alpha_a(T_a - \hat{T}_w) + q_w\right], \tag{24}$$

$$V_{,t} = \omega F_e S_{,t}, \tag{25}$$

$$S_{,t} = \hat{U}_e; \quad m_e U_{e,t} = F_e(P_1 - P_2) - \Sigma P. \tag{26}$$

Conjugation conditions:

$$\hat{P}_n = \hat{P}_0 + \zeta_0 \rho_0 \hat{U}_0^2; \quad \frac{2k}{k-1} R\hat{T}_{gn} = \frac{2k}{k-1} R\hat{T}_{g0} + \hat{U}_0^2; \tag{27}$$

$$\hat{P}_n = \hat{P}_l + \frac{F_l}{F_n} \zeta_l \rho_l \hat{U}_l^2. \tag{28}$$

The structure of the algorithm for the numerical solution of the system of difference equations (16)-(28) is analogous to that set out in [5]. The difference scheme is monotonic, and stable for time space steps of $\Delta t/\tau \leq 0.0005$, $\Delta x/l \geqslant 0.05$.

The algorithm so developed was used in an M-222 computer calculation of the working processes in a cooling system (refrigerator) with a free ejector [4] (steady-state conditions). The machine had the following geometrical and thermophysical characteristics: diameter of the compressor cylinder 0.051 m; piston travel 0.037 m; length, diameter, and travel of the ejector respectively 0.815, 0.025, and 0.018 m; mass of the ejector 0.14 kg; length and diameter of the regenerator 0.076 and 0.013 m respectively; V_{max}/V_{min} ratio of the cold chamber 1.55; duration of the working cycle 0.18 sec (330 rpm); refrigerating efficiency 9 W; working gas helium. The heat-transfer and resistance coefficients in the cooling system and regenerator, the heat-transfer coefficient in the chambers, the thermophysical properties of the working gas, and the materials of the machine parts were taken in conformity with [8-11].

The computing scheme may be expressed in the form of a series arrangement of three chambers (V_1, compressor; V_2, V_3, hot and cold) and the pipes connecting them (water cooler and regenerator). In each time layer the cross flow of gas is calculated successively: from V_1 into V_2, then from V_2 into V_3, and so on. The initial distribution of the thermal gas-dynamic parameters in the pipelines was regarded as linear. The results of the cyclical change in the parameters in various parts of the machine are shown in Figs. 2-4.

When the volume of the compressor chamber 2 (Fig. 1) is reduced, the gas passes through the cooling system 3 into the hot chamber 4. From the chamber 4 the gas passes through the regenerator 5, situated in the main body of the ejector 6, into the cold chamber 7. The hydraulic resistance of the regenerator packing leads to the appearance of a gas pressure difference at the ends of the ejector (approximately 0.08 MN/m^2, Fig. 2), which is sufficient to move the ejector in the direction of the gas flow (Fig. 4, curves 1 and 5). In the calculation the frictional force in the ejector packing was 70 N. The gas temperature and pressure increase in all elements of the machine (Fig. 3).

The increase in the volume of the compressor chamber displaces the ejector to the hot end; the temperature and pressure of the gas in the machine fall, even in the cold chamber, in which the heat-emitting object is being cooled (Fig. 1). The change in gas velocity within the various components during the working cycle is indicated in Fig. 4. In these calculations we assumed that there was no cross flow of gas between the hot and cold zones through the ejector packing.

The proposed algorithm for the numerical solution of the system of differential equations enables us to calculate the thermal gasdynamic and kinetic characteristics of technological systems characterized by the existence of heat transfer between the transient gas flows and the packing of a regenerator situated in freely-moving elements.

NOTATION

c, specific heat of the material; c_r, specific heat of the gas at constant pressure; c_V, specific heat of the gas at constant volume; d, diameter; e, internal energy; F, cross-sectional area of the flow; F_w, heat-transfer surface (wall); F_e, area of the bottom of the ejector; G, rate of gas flow; f, cross-sectional area of the valve; i, enthalpy; I, total enthalpy; k, adiabatic index; m, mass of gas; m_e, mass of ejector; l, length of pipeline; P, gas pressure; p, porosity; q, thermal flux density; R, gas constant; S, displacement of ejector; T, temperature; t, time; U, velocity; V, volume; x, spatial coordinate; α, heat-transfer coefficient; α, β_1, coefficients allowing for the nonuniformity of velocity and density distribution over the cross section of the pipeline; δ, wall thickness; ξ, hydraulic resistance; ρ, density; λ, thermal conductivity. Indices: a, surrounding medium (ambient); e, ejector; h, hydraulic; g, gas; l, end point of the pipeline; r, receiver; p, packing; w, wall; T, tube (pipe); 0, initial; n, number of chamber; c, cylinder.

$$y = y_j^i = y(t, x);\quad \hat{y} = y_j^{i+1} = y(t + \Delta t, x);\quad \hat{y}_{-1} = y_{j-1}^{i+1} = y(t+\Delta t, x - \Delta x);$$

$$y_{,t} = (y_j^{i+1} - y_j^i)/\Delta t = (\hat{y} - y)/\Delta t;\quad y_{,x} = (y_j^{i+1} - y_{j-1}^{i+1})/\Delta x = (\hat{y} - \hat{y}_{-1})/\Delta x.$$

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